Inhomogeneous Coupled-Line Filters with Large Mode-Velocity Ratios

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Abstract—**Experimental and theoretical results are presented demonstrating some novel properties of a new class of filters based on inhomogeneous coupled-line structures having large even–odd mode-velocity ratios, e.g., a single inhomogeneous coupled-line filter section with an equiripple, 3-peak stopband and passband response.**

INTRODUCTION

COUPLED-LINE structures are utilized extensively as building blocks for filters, directional couplers, and other important transmission-line devices [1], [2]. Odd and even propagation mode components [3] are commonly used in analyzing coupled lines. The phase velocities associated with the odd and even modes are equal for lines in a homogeneous medium but unequal for lines in an inhomogeneous medium. Coupled lines in microstrip [4] and suspended-substrate stripline [5] yield small deviations from equal velocities (velocity ratios typically less than 1.2). Much larger deviations can be obtained with inhomogeneous broadside-coupled strips [6] (velocity ratios of 3.0 and even larger). Velocity ratios of roughly 2 to 1 and higher can lead to dramatic useful changes in the performance of familiar circuits.

The purpose of this paper is to present experimental and theoretical results demonstrating some of the novel properties of a new class of filters based on inhomogeneous coupled-line structures having large even–odd mode-velocity ratios. For example, it will be shown that a rippled pass- and stopband response can be realized with a single inhomogeneous section producing a response similar to that of several homogeneous sections in cascade. The devices described are characterized by the desirable feature of complex electrical response produced by a simple physical structure.

ANALYSIS

The inhomogeneous counterpart of each of the ten two-port coupled-line configurations identified by Jones and Bollajahn [7] has been studied. Each configuration exhibits interesting properties not possessed by the homogeneous version. For the purposes of this paper, attention will be focused on the symmetric configurations as shown schematically in Fig. 1. As an example of the performance

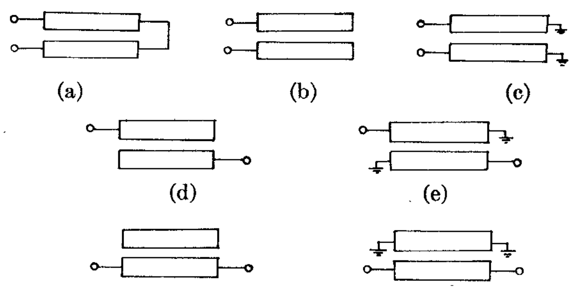

Fig. 1. Symmetrical coupled-line filter sections. (a) C section. (b) Open-circuit comb. (c) Short-circuit comb. (d) Open-circuit digital. (e) Short-circuit digital. (f) Open-circuit symmetrical. (g) Short-circuit symmetrical.

changes that take place when large coupled mode-velocity differences are introduced into an otherwise familiar circuit configuration, consider the microwave C section. In the homogeneous case of equal mode velocities a C section is an all-pass network. As shown below and illustrated in Fig. 2, an inhomogeneous C section with a large even–odd mode-velocity ratio becomes an excellent bandpass network.

Design relations for the inhomogeneous coupled-line filters were obtained by analytically manipulating the insertion-loss expression as given in terms of the $ABCD$ parameters of the network and the generator and load impedances. The $ABCD$ parameters of a pair of inhomogeneous coupled lines can be determined by the method of Jones and Bollajahn [7]. Results for symmetrical lines are given by Zysman and Johnson [8].

General equations for pole and zero locations, relative maxima and minima locations, and insertion loss are given in Table I for the symmetrical filter sections. The symbols

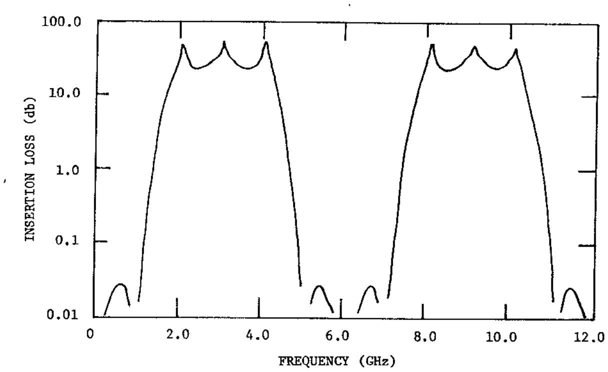

Fig. 2. Response of an inhomogeneous C section with $k = 2$, $M = 3$, $R = 3$, $v_e = 2.28 \times 10^{10}$ cm/s, $l = 1.875$ cm.

Manuscript received May 3, 1974; revised August 29, 1974. This work was sponsored by the National Science Foundation under Grant GK-38117.
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TABLE I
GENERAL DESIGN RELATIONS FOR SYMMETRICAL INHOMOGENEOUS COUPLED-LINE FILTERS

Filter Type	Zero Locations	Pole Locations	Relative Maxima and Minima Locations	Insertion Loss Expression $[\lvert V_2'/V_2\rvert^2 - 1]$
C-Section	$[M-1]\sin(k+1)\theta_e - [M+1]\sin(k-1)\theta_e = 0$	$[R-1]\cos(k+1)\theta_e + [R+1]\cos(k-1)\theta_e = 0$	$[1-RM]\sin^2 k\theta_e + k[R-M]\sin^2\theta_e + R[M-k] = 0$	$T^2\left[\frac{M\sin\theta_e\cos(k\theta_e) - \cos\theta_e\sin(k\theta_e)}{R\cos\theta_e\cos(k\theta_e) + \sin\theta_e\cos(k\theta_e)}\right]^2$
	where for C-Section $M = Z_o^2/Z_{oo}Z_{oe}$, $R = Z_{oe}/Z_{oo}$, $T = Z_{oe}/Z_o$			
Short- and Open-Circuit Comb Section	$[Q-1]\cos(k+1)\theta_e + [Q+1]\cos(k-1)\theta_e = 0$	$[R-1]\sin(k+1)\theta_e + [R+1]\sin(k-1)\theta_e = 0$	$Q[\cos^2 k\theta_e - kR\cos^2\theta_e] + R\sin^2 k\theta_e - k\sin^2\theta_e = 0$	$W^2\left[\frac{Q\cos\theta_e\cos(k\theta_e) + \sin\theta_e\sin(k\theta_e)}{R\cos\theta_e\sin(k\theta_e) - \sin\theta_e\cos(k\theta_e)}\right]^2$
	where for short-circuit comb section $Q = Y_{oe}Y_{oo}/Y_o^2$, $R = Y_{oe}/Y_{oo}$, $W = Y_o/Y_{oo}$		and for open-circuit comb section $Q = Z_{oe}Z_{oo}/Z_o^2$, $R = Z_{oe}/Z_{oo}$, $W = Z_o/Z_{oo}$	
Short- and Open-Circuit Digital Section	$[P-Q]\cos(k-1)\theta_e - [P+Q]\cos(k+1)\theta_e = 2Q$	$R\sin(k\theta_e) - \sin\theta_e = 0$	$\cos\theta_e[PR\sin^2 k\theta_e - kQ\sin\theta_e\sin(k\theta_e) + Q(kR-1)] + \cos(k\theta_e)[-kP\sin^2 k\theta_e + QR\sin\theta_e\sin(k\theta_e) + Q(kR-1)] = 0$	$\frac{W^2}{4}\left[\frac{P\sin\theta_e\sin(k\theta_e) - Q[1+\cos\theta_e\cos(k\theta_e)]}{R\sin(k\theta_e) - \sin\theta_e}\right]^2$
	where for short-circuit digital section $P = [-4Y_o^2+Y_{oo}^2+Y_{oe}^2]/2Y_o^2$, $Q = Y_{oe}Y_{oo}/Y_o^2$, $R = Y_{oe}/Y_{oo}$, $W = Y_o/Y_{oo}$		and for open-circuit digital section $P = [-4Z_o^2+Z_{oo}^2+Z_{oe}^2]/2Z_o^2$, $Q = Z_{oe}Z_{oo}/Z_o^2$, $R = Z_{oe}/Z_{oo}$, $W = Z_o/Z_{oo}$	
Short- and Open-Circuit Symmetrical Section	$[P-Q]\cos(k-1)\theta_e - [P+Q]\cos(k+1)\theta_e = -2Q$	$R\sin(k\theta_e) + \sin\theta_e = 0$	$\cos\theta_e[PR\sin^2 k\theta_e + kQ\sin\theta_e\sin(k\theta_e) + Q(kR-1) + \cos k\theta_e[kP\sin^2 k\theta_e + QR\sin\theta_e\sin(k\theta_e) + Q(1-kR)] = 0$	$\frac{W^2}{4}\left[\frac{P\sin\theta_e\sin(k\theta_e) + Q[1-\cos\theta_e\cos(k\theta_e)]}{R\sin(k\theta_e) + \sin\theta_e}\right]^2$
	where for short-circuit symmetrical section P,Q,R,W are same as for short-circuit digital section		and for open-circuit symmetrical section P,Q,R,W are same as for open-circuit digital section	

TABLE II
DESIGN RELATIONS FOR SPECIFIC VALUES OF k

Filter Type and k	Zero Locations	Pole Locations	Relative Maxima and Minima Locations	Insertion Loss Expression at Location of Relative Max & Min $[\lvert V_2'/V_2\rvert^2 - 1]$
C-Section $k = 2$	$\theta_e = 0°, 180°, \ldots$ and $\theta_e = \sin^{-1}\left(\pm\sqrt{\frac{M-2}{2M-2}}\right)$ for $M \geq 2$	$\theta_e = 90°, 270°, \ldots$ and $\theta_e = \cos^{-1}\left(\pm\sqrt{\frac{R-2}{2R-2}}\right)$ for $R \geq 2$	$\theta_e = \sin^{-1}(\pm\sqrt{x})$ for $x \leq 1$ where $x = [-B \pm \sqrt{B^2-4AC}]/2A$ and $A = 4[RM-1]$ $B = 2[R-M] + 4[1-RM]$ $C = R[M-2]$	$\frac{R}{M}\left[\frac{\sqrt{x}(2x-2Mx+M-2)}{\sqrt{1-x}(2x-2Rx+R)}\right]^2$
Short- and Open-Circuit Comb Section $k = 2$	$\theta_e = 90°, 270°, \ldots$ and $\theta_e = \sin^{-1}\left(\pm\sqrt{\frac{Q}{2Q-2}}\right)$ for $Q > 2$	$\theta_e = 0°, 180°, \ldots$ and $\theta_e = \cos^{-1}\left(\pm\sqrt{\frac{1}{2-2R}}\right)$ for $0 \leq R \leq \frac{1}{2}$	$\theta_e = \sin^{-1}(\pm\sqrt{x})$ for $x \leq 1$ where $x = [-B \pm \sqrt{B^2-4AC}]/2A$ and $A = 4[Q-R]$ $B = 4[R-Q] + k[RQ-1]$ $C = Q-kRQ$	$W^2\left[\frac{\sqrt{1-x}(2x-2Qx+Q)}{\sqrt{x}(2x-2Rx+2R-1)}\right]^2$
Short- and Open-Circuit Digital Section $k = 3$	$\theta_e = \sin^{-1}(\pm\sqrt{x})$ for $x \leq 1$ where $x = [-B \pm \sqrt{B^2-4AC}]/2A$ and $A = 4[P+Q]$ $B = -[3P+5Q]$ $C = 2Q$ NOTE: If $P = -Q$, then zeros occur for $\theta_e = 90°, 270°, \ldots$	$\theta_e = 0°, 180°, \ldots$ and $\theta_e = \sin^{-1}\left(\pm\sqrt{\frac{3R-1}{4R}}\right)$ for $R \geq \frac{1}{3}$	$\theta_e = \sin^{-1}(\pm\sqrt{x})$ for $x \leq 1$ and x is the solution of $Ax^3 + Bx^2 + Cx + D = 0$ where $A = -16R[P+Q]$ $B = 24PR+16QR-12P-12Q$ $C = -9PR+9QR+3P+5Q$ $D = -6QR+2Q$	$\frac{W^2}{4}\left[\frac{-4(P+Q)x^2 + (3P+5Q)x-2Q}{\sqrt{x}(-4Rx+3R-1)}\right]^2$
Short- and Open-Circuit Symmetrical Section $k = 3$	$\theta_e = 0°, 180°, \ldots$ and $\theta_e = \sin^{-1}\left(\pm\sqrt{\frac{3P+5Q}{4(P+Q)}}\right)$	$\theta_e = \sin^{-1}\left(\pm\sqrt{\frac{3R+1}{4R}}\right)$ for $R \geq 1$	$\theta_e = 90°, 270°, \ldots$ and $\theta_e = \sin^{-1}(\pm\sqrt{x})$ for $x \leq 1$ where $x = [-B \pm \sqrt{B^2-4AC}]/2A$ and $A = -16R(P+Q)$ $B = 24PR+16QR+12P+12Q$ $C = -[9PR+15QR+3P+5Q]$	$\frac{W^2}{4}\left[\frac{-4(P+Q)x^2 + (3P+5Q)x}{\sqrt{x}(-4Rx+3R+1)}\right]^2$

Note: M, P, Q, R, W are defined in Table I.

defined in Table I are used in the remainder of the paper. In addition the following definitions are required.

Z_{0e}	Even mode impedance.
Z_{0o}	Odd mode impedance.
v_e	Even mode velocity.
v_o	Odd mode velocity.
l	Physical length of coupled section.
Z_o	Characteristic impedance of input and output lines.
k	$= v_e/v_o$.
$\theta_e = 2\pi fl/v_e$	Even mode electrical length.
$\theta_o = 2\pi fl/v_o$	Odd mode electrical length $= k\theta_e$.
Insertion loss (dB)	$= 10 \log_{10} \lvert(V_2'/V_2)\rvert^2$.

The equations given in Table I can be solved to produce filter-design data for any arbitrary mode-velocity ratio k. Detailed design data have been derived for inhomogeneous coupled-line filters with even–odd velocity ratios $k = \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 2, 3, 4$. The general features of the design equations are illustrated by the selected cases given in Table II. Locations of poles, zeros, and relative maxima and minima are specified plus the values of attenuation at the relative maxima and minima. Plots of these relationships are revealing and useful in examining design possibilities.

Consider, for example, the C-section design graphs for $k = 2$ given in Figs. 3 and 4. Since the insertion-loss properties are periodic with period 180 *even mode* electrical degrees, only a 180° range of θ_e is shown. Notice that zeros are always located at $\theta_e = 0°, 180°$. These locations correspond to passband centers. For $M > 2$ an additional pair of zeros is present in each 180° period of θ_e yielding a wider passband with ripples and a steeper transition between passband and stopband. The position of this second pair of zeros is controlled by adjusting $M = Z_0^2/Z_{0e}Z_{0o}$. For example, if $M = 3$, zeros are located at $\theta_e = 0°, 30°, 150°, 180°$, etc.

Similarly, a pole of insertion loss is always located at $\theta_e = 90°$. For $R > 2$ an additional pair of zeros is present in each 180° period of θ_e yielding a wider stopband with ripples and a steeper transition between passband and stopband. The position of this second pair of poles is controlled by adjusting $R = Z_{0e}/Z_{0o}$. Since the M control of the secondary zeros is independent of the R control of the secondary poles, the two effects can be combined producing both an equirippled passband and stopband similar to what one expects from elliptic filters. Note, however, that the inhomogeneous filter consists of just one physically simple section of coupled line. If $R = 3$, the inhomogeneous C section with $k = 2$ will have poles at $\theta_e = 60°, 90°, 120°$.

Finite, nonzero relative maxima and/or minima of insertion loss are present if $M \geq 2, R > 2$. These locations are functions of both R and M. For example, with $M = 3$, $R = 3$ relative minima are located in the stopband at $\theta_e = 65.2°, 114.8°$, while relative maxima are located in the passband at $\theta_e = 5.3°, 174.7°$. The peak value of attenuation in the passband is the value at each relative maxima which for $M = 3$, $R = 3$ is 0.025 dB. The minimum value of attenuation in the stopband is the value at each relative minima which for $M = 3$, $R = 3$ is 22.4 dB.

The overall calculated response of an inhomogeneous C section with $k = 2$, $M = 3$, $R = 3$, $Z_0 = 50\,\Omega$, $l = 1.875$ cm, $v_o = 1.14 \times 10^{10}$ cm/s is given in Fig. 1. $\theta_e = 90°$ corresponds to approximately 3 GHz. Both an equirippled passband and stopband are present in this single-section filter and the minimum attenuation in the 3-peaked stopband is 22.4 dB.

Special techniques for designing multisection inhomogeneous coupled-line filters are being developed and will be published. The $ABCD$ parameters cited above can be used to analyze a multisection structure if one desires.

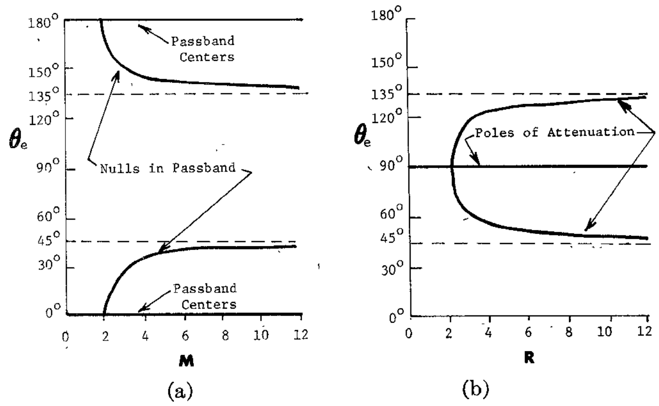

Fig. 3. (a) Locations of zeros of insertion loss for inhomogeneous C sections with $k = 2$. (b) Locations of poles of insertion loss for inhomogeneous C sections with $k = 2$.

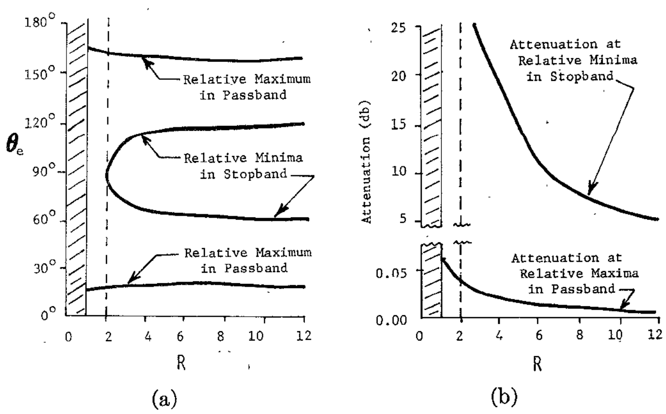

Fig. 4. (a) Locations of relative maxima and minima for inhomogeneous C sections with $k = 2$, $M = 3$. (b) Values of attenuation at locations of relative maxima and minima for inhomogeneous C sections with $k = 2$, $M = 3$.

CALCULATED AND EXPERIMENTAL RESULTS

As mentioned in the Introduction, one structure well-suited to the realization of large differences in odd and even mode-propagation velocities is the broadside-coupled strips in a layered dielectric medium [6] shown in Fig. 5(a). For this structure, the even mode energy is concentrated in the regions above and below the center strips, while the odd mode energy is concentrated in the dielectric between the two center strips. This means that the ratio of even-to-odd mode velocity is approximately $(\epsilon_2/\epsilon_1)^{1/2}$, but fringe-field effects reduce the velocity ratio somewhat, especially for narrow-center conductors. The modified structure [9] shown in Fig. 5(b) permits higher velocity ratios to be realized for a given ratio of ϵ_2 to ϵ_1. It is important to note that proper choice of dielectric constants and dimensions permit $k = v_e/v_o$ to be either less than or greater than one. A wide range of odd and even mode impedances are possible.

Using broadside-coupled strips in an inhomogeneous medium a large variety of unequal mode-velocity devices can be fabricated. The physical structure is very simple, a pair of lines coupled through a substrate. Various terminal conditions, physical dimensions, and dielectric constants lead to the different device types.

Fig. 6 shows the response of an unequal-velocity C-section filter with $k = 2$ and illustrates the positioning of zeros in the passband to increase the bandwidth and the steepness of the transition between passband and stopband. The dotted-line response was obtained by adjusting the values of R and M to position a zero of attenuation in

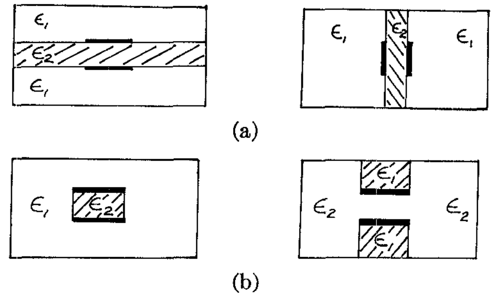

Fig. 5. (a) Inhomogeneous broadside-coupled strips. (b) Modified inhomogeneous broadside-coupled strips.

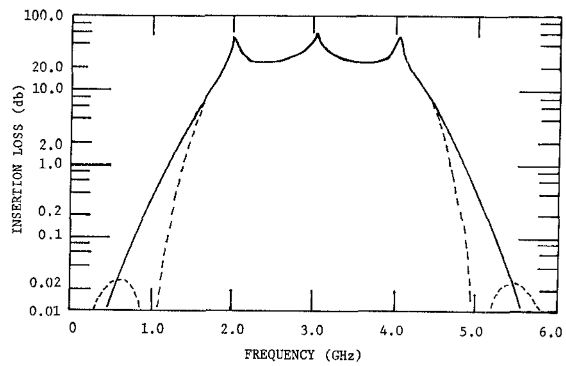

Fig. 6. Comparison of "skirt" steepness of two inhomogeneous C-section filters. Dotted lines correspond to $k = 2$, $Z_{0e} = 67.1\ \Omega$, $Z_{0o} = 22.4\ \Omega$, $Z_0 = 50\ \Omega$, $v_e = 2.28 \times 10^{10}$ cm/s, $l = 1.875$ cm. Solid line corresponds to same parameters, except $Z_{0e} = 50\ \Omega$, $Z_{0o} = 16.7\ \Omega$.

the passband at 1 GHz while maintaining the stopband pole positions and minimum stopband attenuation fixed. By accepting a decrease in minimum stopband attenuation from 22.4 to 19.1 dB and an increase in maximum passband attenuation from 0.025 to 0.65 dB, the transition region can be made much steeper by positioning ($M = 10$, $R = 4$) stopband poles at 1.85, 3.04, 4.23 GHz, and a passband zero at 1.38 GHz.

Figs. 7 and 8 compare the measured and calculated responses of an inhomogeneous C-section filter with $k = 2$. The test structures were designed using the impedance and velocity data for broadside-coupled strips given by Allen and Estes [6]. Both structures were fabricated on a $1 \times 2 \times 0.025$-in A1995 substrate. A suspended substrate microstrip [5] was used for input and output lines. The filter of Fig. 7 has $k = 2.0$ and exhibits 3-peak stopband response with excellent agreement between measured and calculated response. The calculated data assume the lossless case while the measured data naturally show the rounding effect of finite Q's. Fig. 8 illustrates the effect of a large (20-percent) deviation from an integral velocity ratio. The positions of the poles shift and the minimum attenuation in the stopband is decreased.

Figs. 9–11 illustrate responses for 3 other types of inhomogeneous filter sections. First, the short-circuit comb section with $k = 2$, next the open-circuit symmetric section with $k = 3$, and finally the open-circuit digital section with $k = \frac{1}{3}$. It is tempting to say that these are typical responses. However, the response shape, whether it is

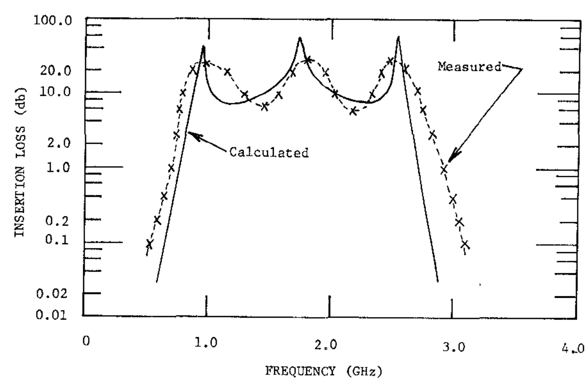

Fig. 7. Comparison of measured and theoretical response for an inhomogeneous C section with $k = 2$, $Z_{0e} = 94\ \Omega$, $Z_{0o} = 11\ \Omega$, $Z_0 = 50\ \Omega$, $v_e = 2.26 \times 10^{10}$ cm/s, $l = 3.23$ cm.

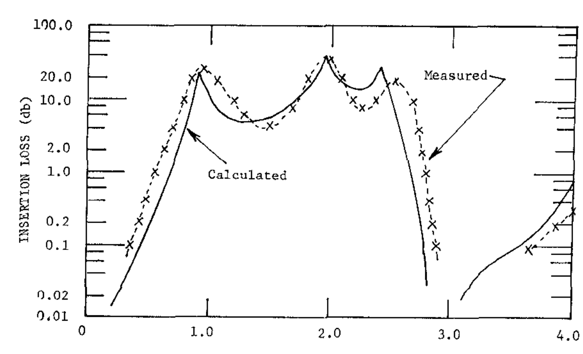

Fig. 8. Comparison of measured and theoretical response for an inhomogeneous C section with $k = 2.2$, $Z_{0e} = 115\ \Omega$, $Z_{0o} = 15\ \Omega$, $Z_0 = 50\ \Omega$, $v_e = 2.43 \times 10^{10}$ cm/s, $l = 3.23$ cm.

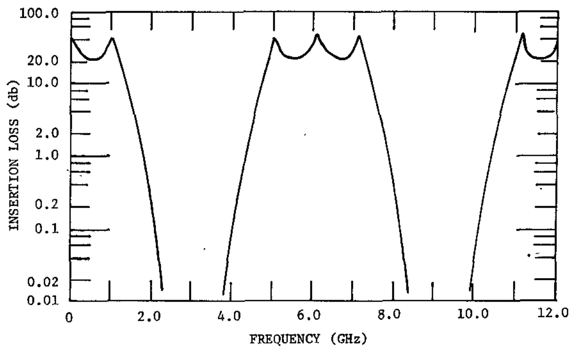

Fig. 9. Response of an inhomogeneous short-circuit comb section with $k = 2$, $Z_{0e} = 60\ \Omega$, $Z_{0o} = 20\ \Omega$, $Z_0 = 50\ \Omega$, $v_e = 2.28 \times 10^{10}$ cm/s, $l = 1.875$ cm.

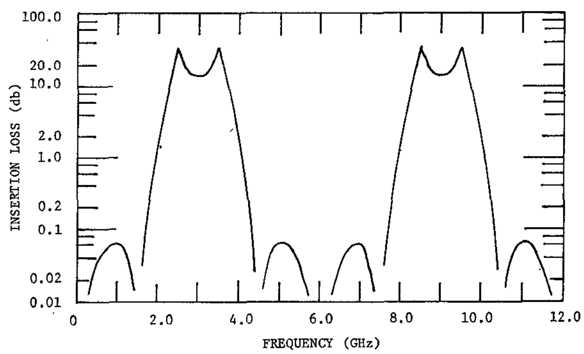

Fig. 10. Response of an inhomogeneous open-circuit symmetric section with $k = 3$, $Z_{0e} = 40\ \Omega$, $Z_{0o} = 30\ \Omega$, $Z_0 = 50\ \Omega$, $v_e = 2.1 \times 10^{10}$ cm/s, $l = 1.75$ cm.

multipole or multizero or both or neither depends upon the velocity ratio, odd and even mode impedances, and input–output line impedances. Many interesting and useful variations are possible.

CONCLUSION

A new class of filters based on inhomogeneous coupled lines with large differences in even and odd mode velocities has been discussed. Inhomogeneous coupled-line filters

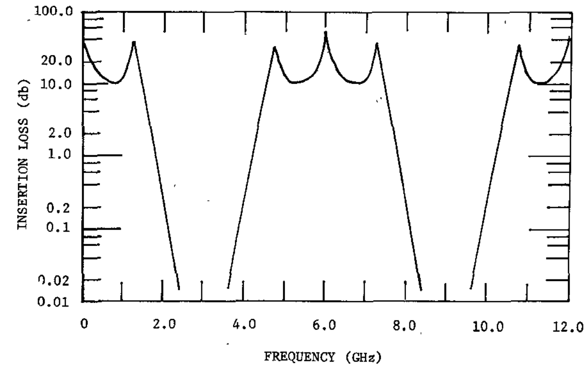

Fig. 11. Response of an inhomogeneous open-circuit digital section with $k = \frac{1}{3}$, $Z_{0e} = 60\ \Omega$, $Z_{0o} = 40\ \Omega$, $Z_0 = 50\ \Omega$, $v_e = 0.7 \times 10^{10}$ cm/s, $l = 1.75$ cm.

have been analyzed, designed, fabricated, and tested. Experimental and theoretical results show such filters to possess desirable and novel properties including both multipole stopband and multizero passband response from a single filter section.

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